LESSON 3.1d

Solving Quadratics by Factoring and Finding the Zeros

Where are we? What are we doing? Where are we going?

Learning how to "***solve***" quadratic functions/equations

… find the "***roots***" of the function/equation … which are the same as the *x*-intercepts.

1. Solve by ***graphing*** the equation on your graphing calculator (yesterday)

2. Solve ***Algebraically***
   - using square roots (L3.1 yesterday)
   - **by factoring / "finding the zeros" of the function (L3.1 today)**
   - completing the square (L3.3)
   - using the quadratic function (L3.4)

Today you will:

- Solve quadratic functions by factoring
- Solve quadratic functions by finding the zeros of the function
- Practice using English to describe math processes and equations

Core Vocabulary:

- zero of a function, p. 96
  - A zero of a function $\boldsymbol{f}$ is an x-value for which $\boldsymbol{f(x) = 0}$ — ...an *x*-intercept / ...a root
- Zero Product Property, p. 96
  - If $a \cdot b = 0$ then what can you say about a and b? One or both are zero.
  - If the product of two expressions is zero, then one or both of the expressions equal zero.
  - If a and b are expressions and $ab = 0$, then $a = 0$ or $b = 0$

Solve $x^2 - 4x = 45$ by factoring.

SOLUTION

$x^2 - 4x = 45$			Write the equation.
$x^2 - 4x - 45 = 0$			Write in standard form.
$(x - 9)(x + 5) = 0$			Factor the polynomial.
$x - 9 = 0$	or	$x + 5 = 0$	Zero-Product Property
$x = 9$	or	$x = -5$	Solve for x.

The solutions are $x = -5$ and $x = 9$.

UNDERSTANDING MATHEMATICAL TERMS

If a real number k is a zero of the function $f(x) = ax^2 + bx + c$, then k is an x-intercept of the graph of the function, and k is also a root of the equation $ax^2 + bx + c = 0$.

You know the x-intercepts of the graph of $f(x) = a(x - p)(x - q)$ are p and q. Because the value of the function is zero when $x = p$ and when $x = q$, the numbers p and q are also called *zeros* of the function. A **zero of a function** f is an x-value for which $f(x) = 0$.

You try it:

Solve $x^2 + 2x = 48$ using factoring and the Zero Product Property

$$x^2 + 2x - 48 = 0$$

$$(x + 8)(x - 6) = 0$$

So either $(x + 8) = 0$ or $(x - 6) = 0$

$$x = -8 \qquad\qquad x = 6$$

The solutions are $x = -8, 6$

Find the zeros of $f(x) = 2x^2 - 11x + 12$.

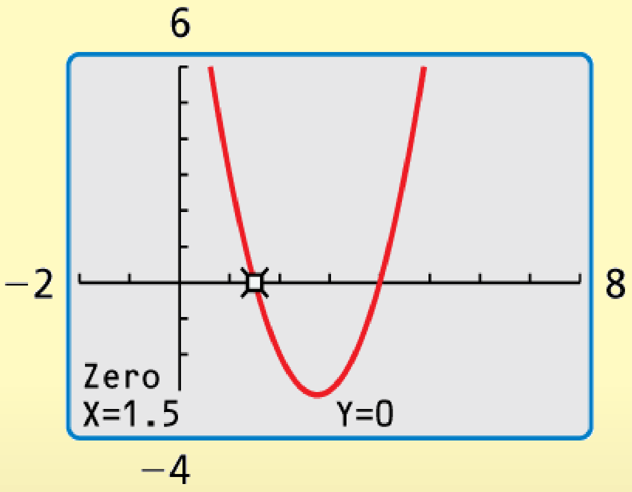
SOLUTION

To find the zeros of the function, find the x-values for which $f(x) = 0$.

$2x^2 - 11x + 12 = 0$				Set $f(x)$ equal to 0.
$(2x - 3)(x - 4) = 0$				Factor the polynomial.
$2x - 3 = 0$	or	$x - 4 = 0$		Zero-Product Property
$x = 1.5$	or	$x = 4$		Solve for x.

The zeros of the function are $x = 1.5$ and $x = 4$. You can check this by graphing the function. The x-intercepts are 1.5 and 4.

Check



For a science competition, students must design a container that prevents an egg from breaking when dropped from a height of 50 feet.

a. Write a function that gives the height h (in feet) of the container after t seconds. How long does the container take to hit the ground?

b. Find and interpret $h(1) - h(1.5)$.

SOLUTION

a. The initial height is 50, so the model is $h = -16t^2 + 50$. Find the zeros of the function.

$h = -16t^2 + 50$ Write the function.

$0 = -16t^2 + 50$ Substitute 0 for h.

$-50 = -16t^2$ Subtract 50 from each side.

$\frac{-50}{-16} = t^2$ Divide each side by -16.

$\pm\sqrt{\frac{50}{16}} = t$ Take square root of each side.

$\pm 1.8 \approx t$ Use a calculator.

Reject the negative solution, -1.8, because time must be positive. The container will fall for about 1.8 seconds before it hits the ground.

INTERPRETING EXPRESSIONS

In the model for the height of a dropped object, the term $-16t^2$ indicates that an object has fallen $16t^2$ feet after t seconds.

b. Find $h(1)$ and $h(1.5)$. These represent the heights after 1 and 1.5 seconds.

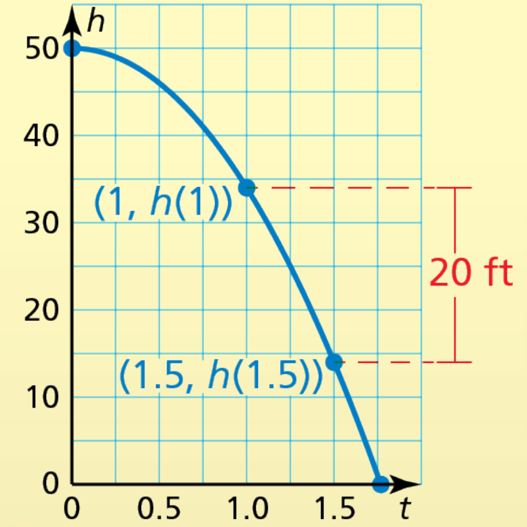
$$h(1) = -16(1)^2 + 50 = -16 + 50 = 34$$

$$h(1.5) = -16(1.5)^2 + 50 = -16(2.25) + 50 = -36 + 50 = 14$$

$$h(1) - h(1.5) = 34 - 14 = 20$$

So, the container fell 20 feet between 1 and 1.5 seconds. You can check this by graphing the function. The points appear to be about 20 feet apart. So, the answer is reasonable.

Check

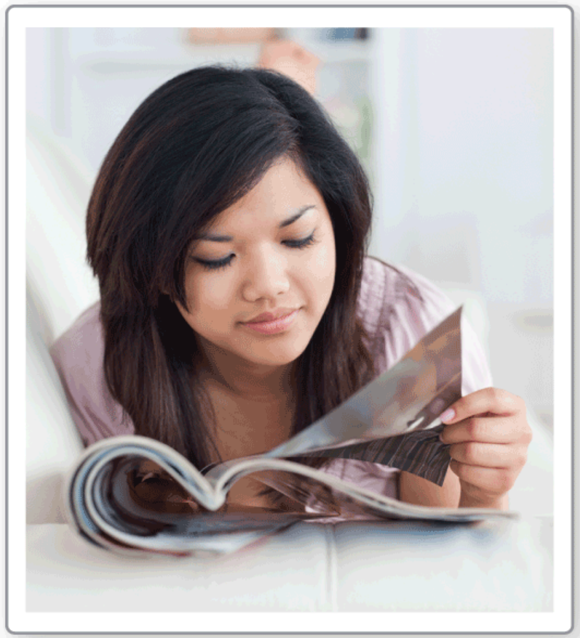


A monthly teen magazine has 48,000 subscribers when it charges \$20 per annual subscription. For each \$1 increase in price, the magazine loses about 2000 subscribers. How much should the magazine charge to maximize annual revenue? What is the maximum annual revenue?

SOLUTION

Step 1 Define the variables. Let x represent the price increase and $R(x)$ represent the annual revenue.

Step 2 Write a verbal model. Then write and simplify a quadratic function.



Annual revenue (dollars) = Number of subscribers (people) • Subscription price (dollars/person)

$$R(x) = (48{,}000 - 2000x) \cdot (20 + x)$$

$$R(x) = (-2000x + 48{,}000)(x + 20)$$

$$R(x) = -2000(x - 24)(x + 20)$$

Step 3 Identify the zeros and find their average. Then find how much each subscription should cost to maximize annual revenue.

The zeros of the revenue function are 24 and -20. The average of the zeros is $\frac{24 + (-20)}{2} = 2$.

To maximize revenue, each subscription should cost $\$20 + \$2 = \$22$.

Step 4 Find the maximum annual revenue.

$$R(2) = -2000(2 - 24)(2 + 20) = \$968{,}000$$

So, the magazine should charge $22 per subscription to maximize annual revenue. The maximum annual revenue is $968,000.

Homework

Pg 100, #27-53 odd, 57, 68